Simplex for Artificial Variables

The program *simplm* solves linear programming problems that may use artificial variables. The method used in the program is the standard simplex procedure.We will explain its use by the following example:

$$\begin{aligned} \text{Minimize } & Z = x_1 + 2x_2 \\ \text{Subject to } & x_1 + x_2 \geq 4 \\ & x_1 - x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

We transform the problem first into a maximization problem, then we subtract surplus variables wherever it may be needed and finally add slack and/or artificial variables as needed. Every time we introduce an artificial variable we have to subtract, in the objective function Z, $\mathbf{M}$ times the variable. In our program, instead of using $\mathbf{M}$, we use $\mathbf{i}$, the imaginary unit. This is very important, otherwise the program will not work. Finally we write all the terms of the objective function on the left hand side. With this modifications, our problem will look like:

$$\begin{aligned} \text{Maximize } \ -Z + x_1 + 2x_2 + 0x_3 + ix_4 + 0x_5 =& 0 \\ \text{Subject to } \ x_1 + x_2 - x_3 + x_4 + 0x_5 =& 4 \\ x_1 - x_2 + 0x_3 + 0x_4 + x_5 =& 2 \\ x_1, x_2, x_3, x_4, x_5 \geq& 0 \end{aligned}$$

The coefficients of the simplex tableau may be put in matrix form as:

$$A = \begin{bmatrix} 1 & 2 & 0 & i & 0 & 0 \\ 1 & 1 & -1 & 1 & 0 & 4 \\ 1 & -1 & 0 & 0 & 1 & 2 \end{bmatrix}$$

We store this matrix in the calculator, say, as a the program call is as follows

$$simplm(a, \ ''out'', it)$$

The first entry is the matrix A as explained above, the second entry is the name of a variable between quotation marks. This variable will contain the solution. If the last entry is 1, the program will show the simplex tableau for every iteration. If we put any other integer as the last entry, the program will avoid showing the intermediate tableaux. For our problem we enter:

$$simplm(a, \ ''o'', 0)$$

After the program shows *done*, we type o and get

$$\{-5 \ \ 3 \ \ 1 \ \ 0 \ \ 0 \ \ 0\}$$

which means $-Z = -5, \ x_1 = 3, \ x_2 = 1, \ x_3 = 0, \ x_4 = 0, \ x_5 = 0.$