Simplex for Artificial Variables

The program *msimplex* solves linear programming problems that may use artificial variables. The method used in the program is the revised simplex procedure.We will explain its use by the following example:

$$\begin{aligned} \text{Minimize} \quad & Z = x_1 + 2x_2 \\ \text{Subject to} \quad & x_1 + x_2 \geq 4 \\ & x_1 - x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

We transform the problem first into a maximization problem, then we subtract surplus variables wherever it may be needed and finally add slack and/or artificial variables as needed. Every time we introduce an artificial variable we have to subtract, in the objective function Z, $\mathbf{M}$ times the variable. In our program, instead of using $\mathbf{M}$, we use $\mathbf{i}$, the imaginary unit. This is very important, otherwise the program will not work. With this modifications, our problem will look like:

$$\begin{aligned} \text{Maximize} \quad & -Z = -x_1 - 2x_2 + 0x_3 - ix_4 + 0x_5 \\ \text{Subject to} \quad & x_1 + x_2 - x_3 + x_4 + 0x_5 = 4 \\ & x_1 - x_2 + 0x_3 + 0x_4 + x_5 = 2 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

In matrix notation this may be written as:

$$\begin{aligned} \text{Maximize} \quad & -Z = cx_S + c_A x_A \\ \text{Subject to} \quad & Ax_S + Ix_A = b \\ & x_S \geq 0, \quad x_A \geq 0 \end{aligned}$$

Where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad c = \begin{bmatrix} -1 & -2 & 0 \end{bmatrix} \quad c_A = \begin{bmatrix} -i & 0 \end{bmatrix} \quad x_S = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad x_A = \begin{bmatrix} x_4 \\ x_5 \end{bmatrix}$$

and I is the 2×2 identity matrix. Notice that the matrices A and c contain the coefficients of the original variables and the surplus variables if any, and I and c_A contain the coefficients of the slack and artificial variables. For the calculator, instead of A we will use a and instead of c_A we use ca. The program call is as follows

$$msimplex(a, b, c, ca, \ ''out'', it)$$

The first four entries are the names of the matrices as explained above, the fifth entry is the name of a variable between quotation marks. This variable will contain the solution. If the last entry is 1, the program will show B^{-1} for every iteration, where B is the matrix of coefficients of the basic variables corresponding to that iteration. If we put any other integer as the last entry, the program will avoid showing B^{-1}. For our problem we enter:

$$msimplex(a, b, c, ca, \ ''o'', 0)$$

After the program shows *done*, we type o and get

$$\{-5 \quad 3 \quad 1 \quad 0 \quad 0 \quad 0\}$$

which means $-Z = -5,\ x_1 = 3,\ x_2 = 1,\ x_3 = 0,\ x_4 = 0,\ x_5 = 0.$