Interior Point

The program *intpt* finds approximations to the solution of a linear programming problem by using the Interior Point Approximation Method. We illustrate its use with the following example

$$\begin{aligned} \text{Minimize } & Z = x_1 + 2x_2 \\ \text{Subject to } & x_1 + x_2 \geq 4 \\ & x_1 - x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Our initial point will be $x_1 = 3,\ x_2 = 2$. Notice that the initial point must satisfy the constraints. We transform the problem first into a maximization problem, then we subtract surplus variables wherever it may be needed and finally add slack variables as needed. With this modifications, our problem will look like:

$$\begin{aligned} \text{Maximize } & -Z = -x_1 - 2x_2 + 0x_3 + 0x_4 \\ \text{Subject to } & x_1 + x_2 - x_3 + 0x_4 = 4 \\ & x_1 - x_2 + 0x_3 + x_4 = 2 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

We use the constraints to compute x_3 and x_4 which give us the following initial point $x_1 = 3,\ x_2 = 2,\ x_3 = 1,\ x_4 = 1$ In matrix notation this may be written as:

$$\begin{aligned} \text{Maximize } & -Z = cx \\ \text{Subject to } & Ax = b \\ & x \geq 0 \end{aligned}$$

Where

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad c = \begin{bmatrix} -1 & -2 & 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad x_0 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

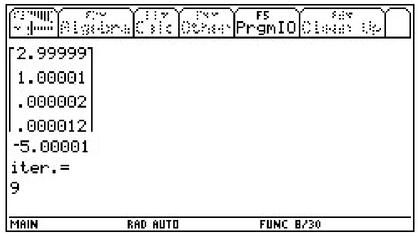
For the calculator, instead of A we will use a. We store in the calculator matrices a, c, x_0. The program call is as follows

$$\mathit{intpt}(a, c, x_0, \alpha, \epsilon, nit)$$

The first three entries are the names of the matrices as explained above, α is a decimal number greater than 0 and less than 1, in our example we will take 0.9. The next entry, ϵ indicates the degree of approximation of the computations, if we take 0.0001 will give us an approximation to the fourth decimal place. The last entry will tell the calculator how many iterations we want to see. For our problem we enter:

$$\mathit{intpt}(a, c, x_0, .9, .0001, 0)$$

After the program shows *done*, it will go the output screen and show in *pause* a vector that contains the values of x_1, x_2, x_3, x_4, after pressing ENTER we will see the value of $-Z$, and pressing ENTER again will give us the number of iterations needed to reach the prescribed approximation. this can be seen in the following screen.



If the last parameter in the calling statement was an integer n different from zero, the program would show the results of the first n iterations.