Chebyshev, version 1.0, 2024-11-30

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Given a finite real intervall [a,b] and a continuous function f:[a,b]->R, the task is to find a good approximation of f (in the maximum norm over [a,b]) by polynomials of degree <N. A good method is to use the Chebyshev polynomials defined on [-1,1] by

or equivalently by recursion on all of R:

If the interval is [-1,1], choose nodal points , and coefficients

1. , *j=0,…,N-1.*

The approximating polynomial of degree N-1 is then

.

I f we start with an arbitrary intervall [a,b], in (1) we have to take

instead of .

The approximating polynomial is then .

The file chebyshev.tns contains three functions cheby1(f,a,b,n),cheby2(a,b,c,m,x) and cheb(n).

On page 1.1 define first the function f you want to approximate, for example

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Then enter c:=cheby1(„f“,-2,2,20) to generate the list c[1….21] of the coefficients c0 … c20 for the approximating polynomial of degree 20 on the interval [-2,2]

On page 1.2 (a graphs page) enter f1(x)=cheby2(-2,2,c,8,x) to have the graph of an approximating polynomial of degree 8 (or any degree <= 20) displayed. Here, c is the list just generated.

The function cheb(n,x) has only demonstrative purpose: it returns Tn(x).