Actuarial Math on the TI-89

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The TI group file, actuary.tig, provides a set of basic functions, such as D, N, S, C, M, and R, for computing the actuarial present values of life insurance and life annuities, as well as premiums and contributions. It assumes a basic working knowledge of actuarial mathematics. The calculations here represent net, not gross, premiums and contributions; i.e. they do not include the expenses of the insurance company or agent commissions, etc.

Each actuarial function allows for calculations based on three different life tables: a primary life (X), a secondary life (Y), and a joint life (J). Mortality for the primary and secondary lives is specified by selecting appropriate mortality tables. A good source of hundreds of mortality tables is the Society of Actuaries Table Manager 3.0 software that can be found at www.soa.org. Simply enter 'table manager' in the search box on the home page and follow the instructions for downloading and using the software. The individual mortality tables can be exported from the Table Manager to a text file. If you cut out all the extraneous text, including the hash value at the bottom of the table, and save only the age and mortality numbers in a text file, you can use the TIDataEditor in TI Connect to import the text file and then save it as a matrix. It can then be transferred to the calculator.

I used four sources in developing the actuarial functions. The first was *Actuarial Mathematics* by Bowers, et al, The Society of Actuaries, 1986. The second was *QLIB/48: Users Guide* by Steve Lindblad, 1997; this is an actuarial library for the HP-48GX calculator. The third was *A Problem-Solving Approach to Pension Funding and Valuation*, 2nd ed., by William H. Aitken. And the fourth was *Introduction to the Mathematics of Demography*, by Robert L. Brown, 1991. Any errors in the concepts or code developed her are, of course, my own! If you find any errors, please let me know at don.phillips@gmail.com.

Setting the Assumptions

The commutation functions are determined by the mortality table(s) you select any age setbacks or set forwards, the interest rate, age difference between the primary and secondary lives, terminal age of the table(s), and the radix. All this information goes into the CommFunc(tables, %I, axmy, Tω, radix).

The arguments are:

tables – a vector or 2-row matrix specifying a primary or a primary/secondary mortality table and any age setbacks or set forwards. Setbacks or forwards allow you to shift the mortality rates up to later ages (setback) or down to earlier ages (set forward). For example, a 2-year setback shifts the mortality rates up two years so that a person 50 years old, for example, will now have the same mortality rate as a person 48 years old under the original table. Setbacks therefore decrease the level of mortality for any given age (if the mortality rates are increasing, as is usually the case). Set forwards, on the other hand, increase the level of mortality for any given age. Setbacks are entered as positive numbers and set forwards as negative numbers. If there is no setback/set forward simply enter a 0.

%i – the assumed interest rate entered as a percentage.

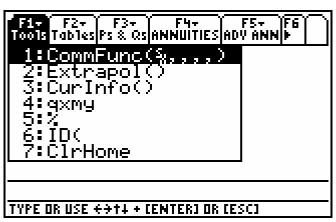
axmy – the age difference between x and y. Subtract the age of Y from the age of X. If the difference is negative enter it as a negative number.

tω – the terminal age truncation option allows you to set an age beyond which the mortality table you are using is not permitted to extend. If you enter an age for tω that is less than the highest age in the mortality table, mortality rates at the end of the table are discarded to force the desired terminal age. This can be useful when you are trying to match commutation functions produced with truncated tables. Thus, if tω is set to 100, the mortality rates above age 99 will be discarded and the mortality rate for age 99 will be set to 1, so that the terminal age of the table is 100. If you do not want to use a truncated table, simply enter tω instead of a number.

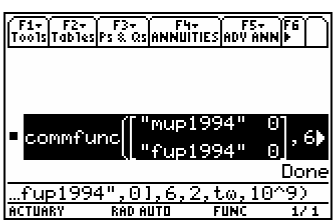
radix – the number of lives assumed to exist at the initial age of the mortality table. I usually use 10^9.

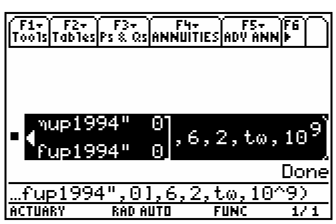
The mortality tables used in the examples presented below are the Male UP-94 and Female UP-94 tables. ('UP' refers to uninsured pensioner.) They are tables #832 and#833 in the Society of Actuaries Table Manager software. The tables are mup1994 and fup1994 in the Actuary folder on the calculator. These two tables are used with no age setbacks or set forward, an interest rate of 6 percent, an age difference of 2, no terminal age truncation, and a radix of 10^9.

To set up the custom menu for the actuarial functions, change to the ACTUARY folder and run the MENU program. Press F1, 1 to access the CommFunc().

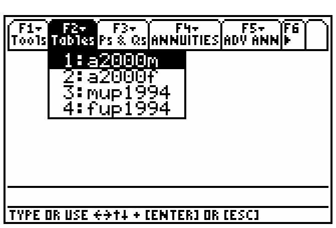


Enter the assumptions:





As noted, the mortality tables and age setbacks are entered in a 2-row matrix. <u>Be sure to enclose the mortality table names in " "</u>. The names of the four mortality tables included with the program are located under F2. You can use them to enter the names in the matrix. If you add other mortality tables to your calculator, you can edit the MENU program to include their names under F2.

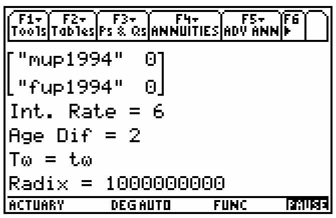


The a2000m and a2000f tables are the annuity tables #864 and #863 in the Society of Actuaries Table Manager software.

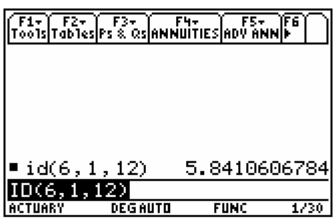
It takes a good three to five minutes to calculate all the commutation functions for X, Y, and J. As the program is running, it displays 'Processing x…', etc., until it is done.

The other functions under F1: Tools are Extrapol(), CurInfo(), and ID(i, nth, mth). The Extrapol() lets you choose between a uniform distribution (udd) and a linear distribution (lin) of deaths between integral ages.

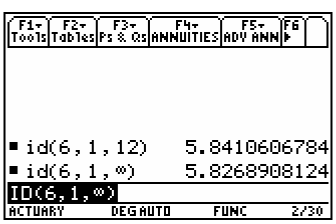
CurInfo() displays a list of the current assumptions. Press ENTER when you are done reviewing the assumptions.



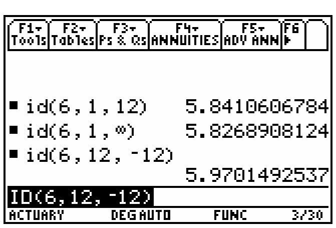
ID(i, nth, mth) converts between interest rates or discount rates compounded nthly to rates compounded mthly. For instance, a 6% annual effective rate of interest is equivalent to 5.84106…% compounded monthly.



A 6% annual effective rate of interest is equivalent to 5.8268…% compounded continuously.



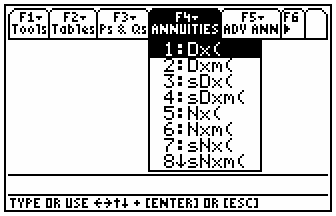
If nth or mth are negative, they refer to a discount rate. A 6% rate compounded monthly is equivalent to a 5.9701…% discount rate compounded monthly.

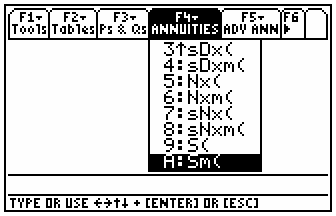


Finally, qxmy returns the age difference between x and y and % is simply the percent function. I just like having it in a convenient place.

1. Life Annuities

The basic commutation functions for life annuities are located under F4: ANNUITIES.





There are 10 commutation functions for basic annuities. They all take at least two arguments: age and person. Person is entered as x, y, or j and is always the last argument entered. The three basic annuity commutation functions are Dx, Nx, and S. These are used to compute annual annuities. Dxm, Nxm, and Sm compute annuities payable mthly, e.g., monthly, quarterly, etc. sDx and sNx are used to compute annuities payable yearly that increase by a given percentage each year; the s stands for a salary scale. And sDxm and sNxm are used to compute annuities payable mthly that increase each year by a given percent.

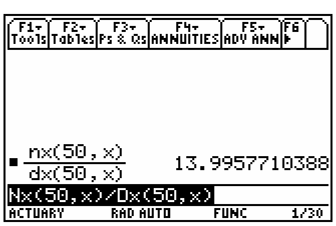
All actuarial calculations that use the basic commutation functions use a ratio of the functions. For instance, an Nx is usually divided by a Dx. And a sNx is usually divided by an sDx. This is just the result of how the commutation functions are set up. The basic annuity commutation function is Dx. It is defined as $\nu^{\wedge x} * l_x$ where ν (Greek nu) is 1/(1 + i) (i is the interest rate) and l_x is the number of persons living at age x based on a mortality table. Using Dx alone in a calculation will give you nothing. It has to be combined in a ratio. For example, suppose you have $(\nu^{\wedge 65} * l_{65})/(\nu^{\wedge 60} * l_{60})$, i.e. Dx(65,x)/Dx(60,x). Now, the ratio of the l's gives you the probability of a person age 60 living to age 65. And the ratio of the ν's gives the present value of 1 to be received in 5 years. The result is the actuarial present value of 1 to be received in 5 years.

The commutation function Nx is the sum of all Dx from age x to the terminal age of the mortality table. And the commutation function S is the sum of all Nx from age x to the terminal age of the mortality table.

The best way to see how they work is through some examples.

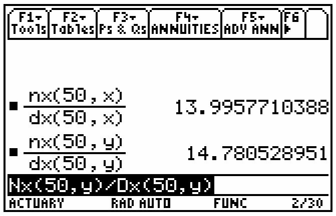
Example 1: Compute the actuarial present value (APV) of a life annuity of 1 payable at the beginning of each year to X age 50. (By stipulating "X age 50" I mean a person age 50 using the primary table of commutation functions.)

The APV of a life annuity of 1 payable at the beginning of the year is given by Nx/Dx, i.e., the N commutation function for age x divided by the D commutation function for age x.

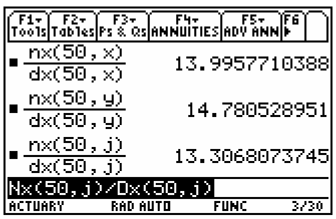


The APV is 13.9957710388. This means that X could pay $14 at age 50 to purchase an annuity where he would receive $1 at the beginning of each year for every year he is alive.

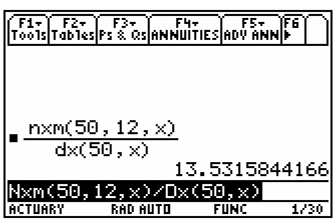
To calculate the same APV for the secondary life (Y), enter:



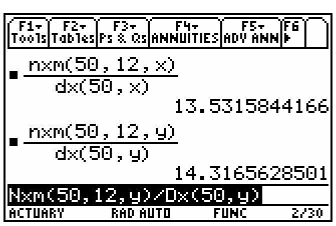
Or, for an annuity payable on the joint life status of X and Y, i.e., as long as both X and Y are alive:



Example 2: Compute the same APV but with the annual annuity of 1 payable monthly. That is, the annuitant receives 1/12 of the annuity each month. For this calculation use the Nxm function instead of the Nx function. In this case, m would equal 12 since the annuity is paid monthly. (m can take on any value from 1 to ∞; ∞ is used for annuities payable continuously.) For X age 50 we have:

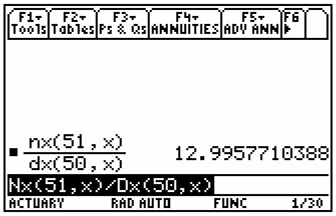


For Y age 50:

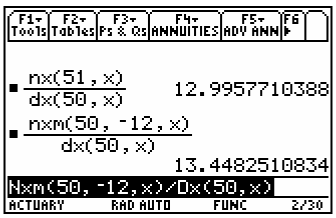


A question: Why does it cost Y more than X for the same annuity? The answer is that Y is female and females on average live longer than males.

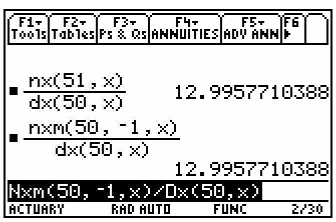
Example 3: Compute the same APVs but with the annuities payable at the end instead of the beginning of the period. For the annuity payable annually, simply increase the age of the Nx commutation function by 1. For X age 50 the APV of the annuity payable at the end of the year is $13.00.



To calculate the APV of annuities payable mthly at the end of the period, enter m as a negative number. For example:

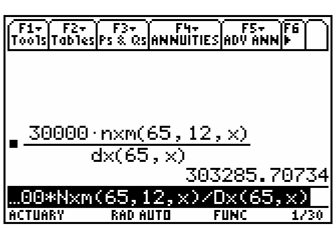


(The fact that a negative payment frequency gives you annuities payable at the end of each period is a consequence of the mathematical definition of the actuarial functions.) The result given by Nx(51,x)/Dx(50,x) can be obtained by setting m to -1 in the Nxm function.

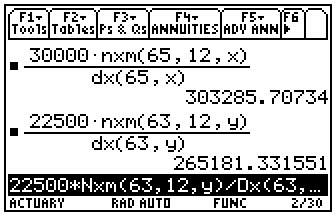


Example 4: Calculate a 75% joint-and-survivor annuity. The J table, along with the X and Y tables can be used to calculate joint-and-survivor (or last-to-die) annuities. Assume X, age 65, receives an annuity of $30,000 a year as long as he lives and his wife Y receives 75% of $30,000 (or $22,500) for as long as she lives after X dies. Assume also that the annuity is payable monthly at the beginning of each month and that Y is 2 years younger than X. (Remember that Y was assumed to be 2 years younger than X when the commutation functions were computed.) Compute the APV of this annuity.

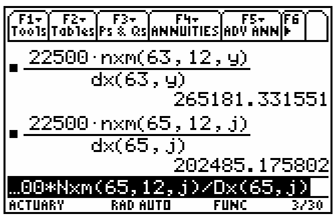
The APV can be computed in three steps. First, X will receive an annuity of $30,000 as long as he lives. The APV is:



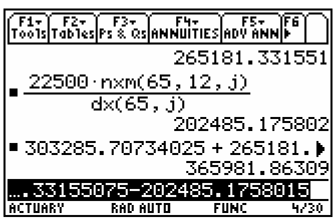
Second, Y will receive an annuity of $22,500 as long as she lives. Since she is 2 years younger than X, enter her age as 63. The APV is:



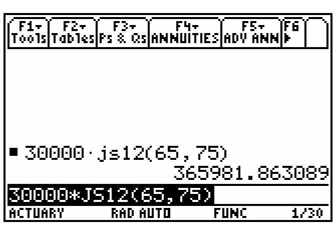
But Y will not receive her annuity as long as both X and Y are alive. From the addition of their annuities' APV, we need to subtract the APV of $22,500 for as long as both are alive. The joint mortality of X and Y is indexed to X's life with Y's age equal to X's minus their age difference. So, when using the joint commutation functions enter only X's age. The APV is:



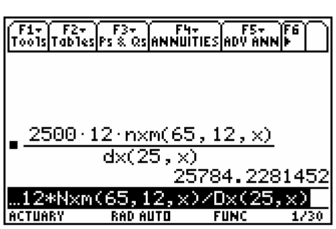
The APV of this 75% joint-and-survivor annuity is therefore:



If X is alive, he receives his annuity of $30,000; Y's annuity of $22,500 is cancelled by the joint annuity of $22,500. If X dies and Y is still living, X's annuity of $30,000 is no longer paid and the joint annuity of $22,500 is no longer viable either; this leaves Y with an annuity of $22,500 for as long as she lives. The same APV can be computed with the JS12(age, percentage) function under F5: ADV ANN.

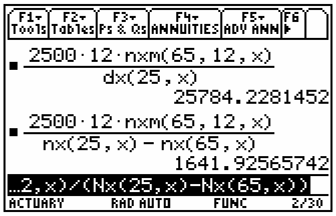


Example 5: Suppose X, age 25, wants to provide for a retirement annuity of $2,500 a month when he retires at age 65. What is the APV of this deferred annuity?

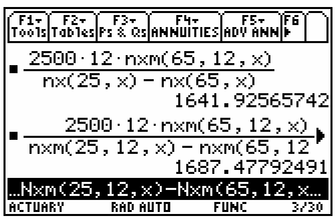


Thus, to receive $2,500 a month for life when he reaches age 65, X would only have to pay $25,784.23 at age 25.

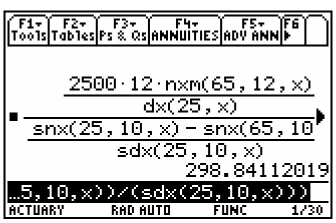
If X wanted to make annual payments to fund his deferred annuity, what would they be?



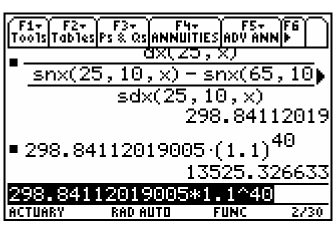
X's contribution to his retirement fund would be $1,641.93 payable at the beginning of each year until he was age 64. Or, X could make an annual contribution of $1,687.48 with 1/12th ($140.62) payable at the beginning of each month until he was age 64 and 11 months.



Now, suppose X wanted to make annual contributions that increased by 10% a year to his deferred annuity. Here we have to use the commutation functions that incorporate a salary scale.

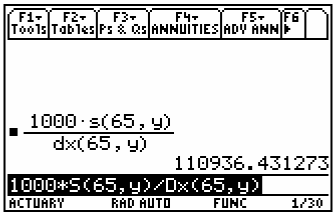


X's contributions begin at $298.84 when he's 25 and end at $13,525.33 when he's 64.



Let's take this calculation apart. First, the 2500*12 is the annual pension or annuity. Second, Nxm(65,12,x)/Dx(25,x) is the APV at age 25 of the pension. Next, (sNx(25,10,x)-sNx(65,10,x))/sDx(25,10,x) is the APV at age 25 of a 40-year term annuity of 1 that increases by 10% a year. Divide the APV of the pension by the APV of the annuity of 1 gives the beginning annual contribution to fund the pension which will increase by 10% each year following the first year.

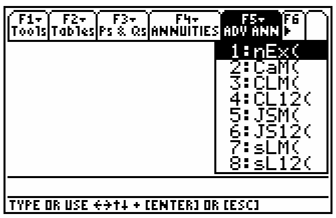
Example 6: What is the APV of an annuity of $1,000 that increases by $1,000 a year if it is purchased at age 65 by Y? For this calculation use the S commutation function.



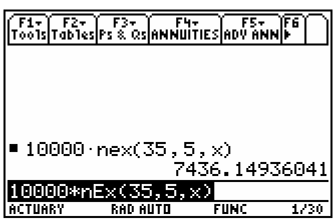
This annuity can be purchased of the single premium of $110,936.43. (The reason for using S instead of Sx for the name of the commutation function is that sx is a reserved name on the TI-89.)

2. Advanced Annuities

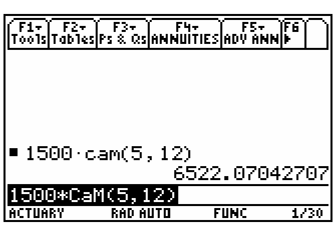
The advanced annuity functions are located under F5: ADV ANN.



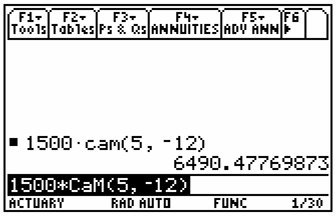
Example 7: nEx(age, n, person) is the endowment function; it is the APV of 1 to be received in n years. What is the APV of a $10,000 endowment to be received by X age 35 in 5 years, if he's still alive?



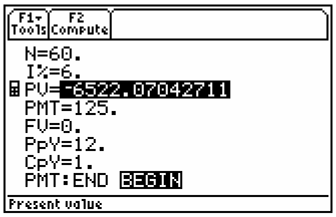
Example 8: CaM(n, mth) is the present value of an annuity due certain of 1 payable mthly. If mth is negative, it is an annuity immediate, not an annuity due. Since this is an annuity certain, it is good for all ages and persons. What is the present value of an annuity due of $1,500 payable monthly for 5 years? The annual interest rate, of course, is 6%.



If this was an annuity immediate, the present value would be:

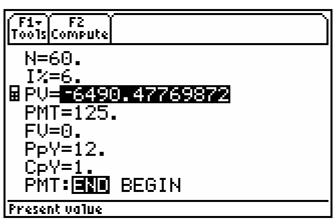


These values can be computed with the Finance App.

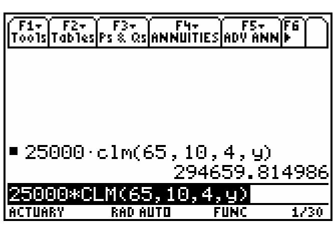


The annual annuity of $1,500 is payable monthly which is $125/month. It is received for 5 years or 60 months. The payments are made at the beginning of the month and the 6% interest rate is compounded annually.

If the payments are made at the end of the month:

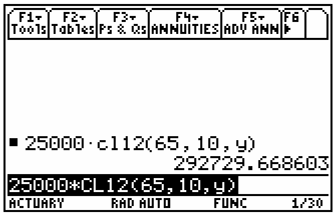


Example 9: CLM(age, n, mth, person) computes the APV of an n-year certain-and-life annuity of 1 payable mthly for a person age x. What is the APV of a 10-year certain-and-life annuity of $25,000 payable quarterly for Y age 65? (In this case Y receives the $25,000 for 10 years, alive or dead, and for the rest of her life after 10 years if alive.)

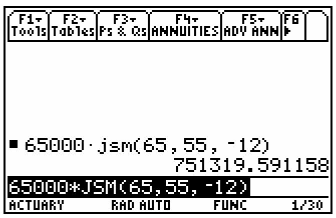


The APV is $294,659.81.

Example 10: CL12(age, n, person) computes the APV of an n-year certain-and-life annuity of 1 payable monthly for a person age x. If the above annuity were payable monthly instead of quarterly, the APV would be $292,729.67.

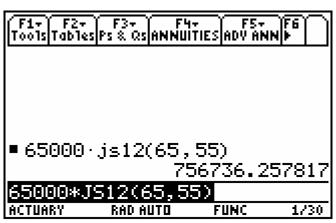


Example 11: JSM(age, percent, mth) computes the APV of a joint-and-survivor annuity payable mthly. It assumes that the annuity belongs to person X and person Y receives a survivor annuity if X dies. Age is the age of X, percent is the percentage of X's annuity Y receives if X dies, and mth is how often the annuity is paid each year. For example, X age 60 retires and receives a pension of $65,000 a year payable monthly at the end of each month. X's partner Y receives 55% of X's pension if X dies. X is 2 years older than Y. What is the APV of the pension?



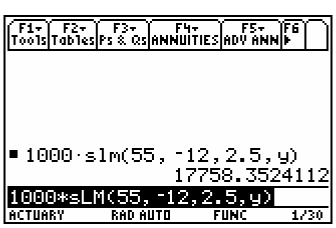
The APV is $751,319.59. Notice that mth is negative. This is because the annuity is paid at the end of the month instead of the beginning of the month.

JS12(age, percent) computes the APV of a joint-and-survivor annuity payable mthly at the beginning of the month. What is the APV of the above pension?

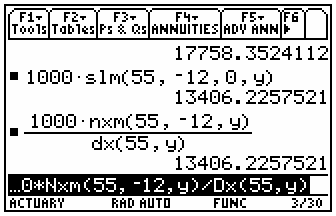


The APV is $756,736.26, over 5,000 more than if it was paid at the end of the month. This is reasonable since X gets his pension sooner rather than later.

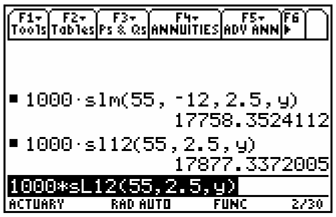
Example 12: sLM(age, mth, scale, person) computes the APV of a life annuity that increases each year by a scale factor and is payable mthly. Y is age 55 and purchases a life annuity of $1,000 that increases by 2.5 percent a year and is payable at the end of each month. What is the APV of this annuity?



The APV is $17,758.35. If there was no scale factor, the APV would be $13,406.23. This can be confirmed with the basic commutation functions.



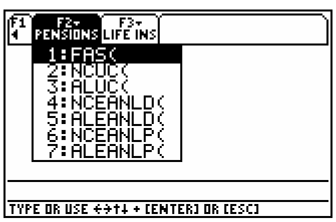
If this annuity was paid monthly at the beginning of each month, the function sL12(age, scale, person) could be used to compute it.



As usual, the APV is a little higher because the payments come at the beginning instead of the end of the month.

3. Pensions

Under F6 are some basic functions for computing the normal cost and actuarial liability of basic pension.

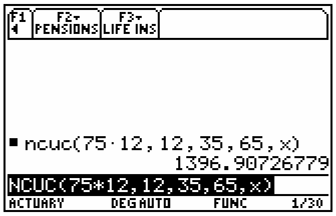


The normal cost (NC) of a pension for any given year is the actuarial value of the part of the total pension benefit assigned to the year following the valuation date, assuming valuation at the beginning of the year. The actuarial liability (AL) is the current value of past normal costs.

Example 13: Traditional Unit Credit (TUC)

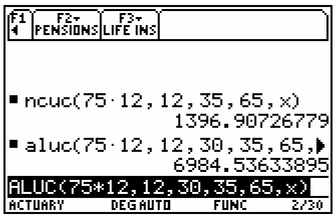
The TUC cost method is usually used with pension plans that provide a flat pension benefit, such as $50 a month per year of service. The normal cost of this type plan is computed with the NCUC(bx, mth, cage, rage, person). ‘bx’ is the annual pension benefit earned in the following year, ‘mth’ is how often the pension is paid, ‘cage’ is the persons current age, ‘rage’ is the expected retirement age, and ‘person’ is X, Y, or J.

X earns a pension benefit of $75 a month for each year of service. If X age 35 expects to retire at age 65, what is the NC of his pension in the following year? What is the pension plan’s actuarial liability if X was hired at age 30?

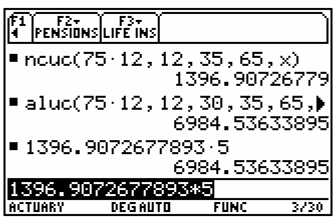


The NC is $1,396.91. Notice that this is the amount that must be put into the pension plan at the beginning of the year to fund the benefit earned during the year. Since the benefit is yet to be earned, there is no liability associated with it until the end of the year.

To find the actuarial liability use the ALUC(bx, mth, eage, cage, rage, person) function where ‘eage’ is the entry age into the pension plan.



The AL is $6,984.54. This is 5 times the NC for X’s 6th year of employment. There is no liability for the 6th year because it has yet to be earned.

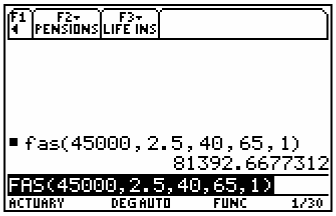


Example 14: Projected Unit Credit (PUC)

The PUC cost method adds the use of a salary scale to the TUC method. The PUC assumes that the pension benefit earned in any year is based on the final salary or final average salary. Final salary is the salary in the year preceding retirement.

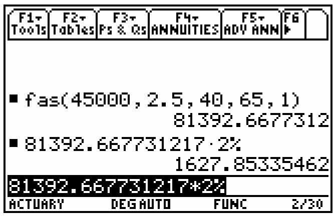
X age 40 earns a pension benefit each year equal to 2% of his final salary. His current salary is $45,000 and is expected to increase by 2.5% a year. He was hired at age 30 and expects to retire at age 65. What is the NC of his benefit earned in the following year and what is the AL?

The NCUC() and ALUC() functions along with the final average salary function, FAS(cs, g, cage, rage, yrs). ‘cs’ is the current salary, ‘g’ is the expected growth rate of the salary, ‘cage’ is the current age, ‘rage’ is the retirement age, and ‘yrs’ is the number of year’s salary to be averaged. Use 1 for ‘yrs’ if only the final salary is to be used.

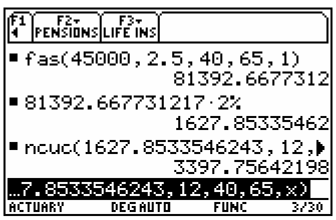


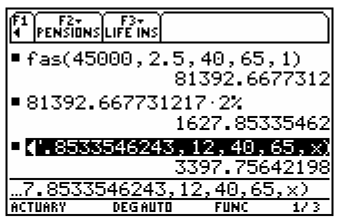
The final salary is $81,392.67.

The pension benefit earned in the following year is 2% of the final salary or $1,627.85.

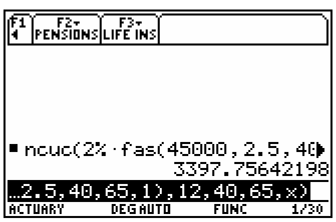


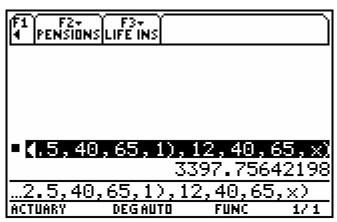
The NC is therefore $3,397.76.



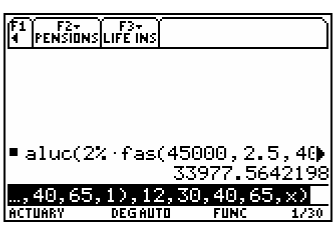


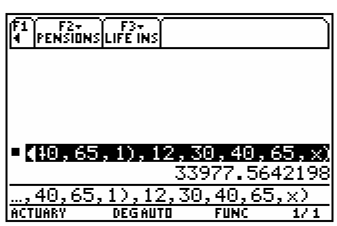
Everything can be put together at once.





The AL is $33, 977.56.



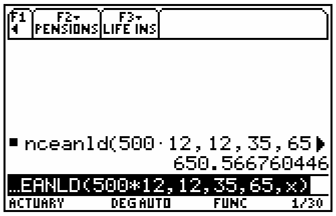


Example 15: Entry Age Normal (EAN) Cost Method

The Entry Age Normal normal cost is such that at age e, the entry age into a pension, the present value of all future normal costs equals the present value of all future benefits. The entry age normal method requires an estimate of a person's full retirement benefit.

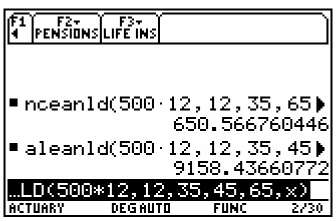
X age 45 plans to retire at age 65 and receive a pension of $500 a month. He was hired at age 35. What are the NC and AL?

To compute the NC use the NCEANLD(br, mth, eage, rate, person) function where eage is the entry age into the pension plan. The LD stands for level dollar; i.e. the normal cost will be the same amount every year (unless assumptions change and then a revised normal cost is computed). (If assumptions change and normal costs are revised, a supplemental liability may be created which may be funded through supplemental costs added to the normal costs.)



The normal cost is $650.57.

ALEANLD(br, mth, eage, cage, rage, person) is used to compute the actuarial liability.

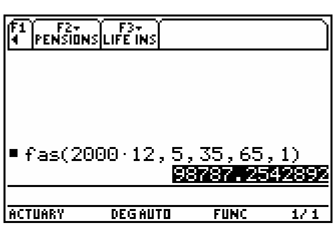


The AL is $9,158.44.

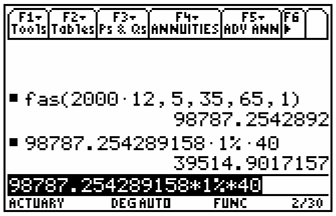
The Entry Age Normal cost method usually uses a salary increase function when the benefit is based on a final salary or final average salary. When salaries are assumed to increase, the normal cost is defined as a level percentage (LP) of salary. When this is the case NCEANLP(br, mth, sc, eage, cage, rage, person) and ALEANLP(br, mth, sc, eage, cage, rage, person) are used to calculate the NC and AL.

X age 35 plans to retire at age 65 and receive 1% of his final monthly pay rate per year of service. His current salary is $2,000 per month and he was hired at age 25. His salary increases 5% each year. What is the normal cost and actuarial liability?

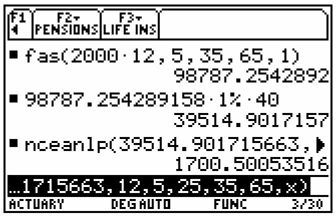
X's final average salary is $98,787.25,



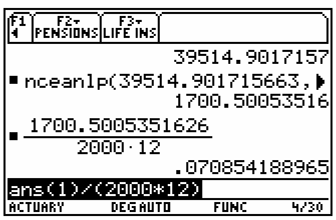
His annual retirement benefit is $39,514.90 (1% of final salary times forth years of service).



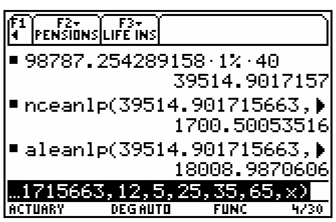
And the NC for the year is $1,700.50. (All the remaining arguments of NCEALP() can be seen on the command line.)



This represents 7.085 percent of his current salary. Assuming salary projections do not change, the NC of future years will be 7.085 percent of the projected salary for that year.

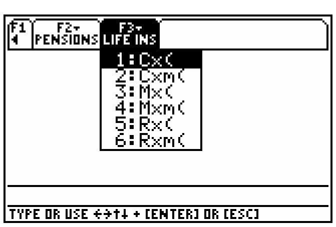


The AL is $18,008.99.



5. Life Insurance

Premiums for life insurance may be calculated with the functions under F6 and then F3: LIFE INS.



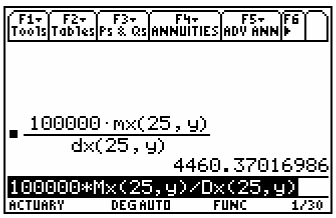
The basic life insurance functions are Cx(age, person), Mx(age, person) and Rx(age, person). Mx() is the sum of Cx()'s from age to the end of the mortality table. And Rx() is the sum of Mx()'s from age to the end of the mortality table.

Cx() is defined as $(l_x - l_{x+1})*v^{x+1}$, where l_x is the number of persons living at age x. Cx() therefore represents the number of persons dying from age x to age x + 1 times an interest rate factor evaluated at age x + 1. It is used to calculate the value of life insurance paid at the end of the year of death. Cx() is rarely used in life insurance calculations because most people buy more than just a one-year term insurance. Mx() is mostly used.

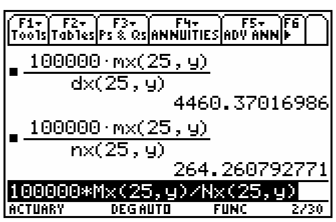
Example 16: Whole Life Insurance

What is the APV or net single premium of a whole life insurance policy of $100,000 payable at the end of the year of death for Y age 25? The APV of a whole life policy is given by Mx/Dx.

The APV of the policy is $4,460.37.

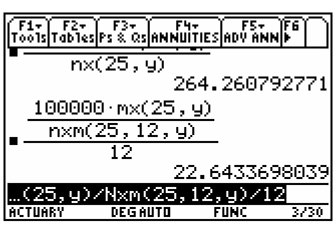


If Y wanted to pay annual premiums on this policy at the beginning of each year for as long as she lives, what would they be?



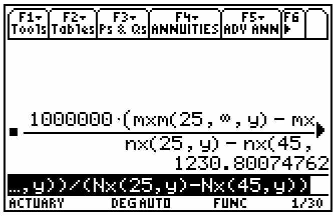
The annual premiums would be $264.26. Notice that Mx is divided by Nx where Nx represents a life annuity.

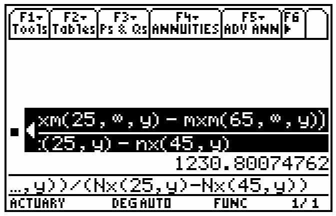
Monthly premiums payable at the beginning of each period would be $22.64.



Example 17: Term Insurance

Y want to insure her life for $1,000,000 during her working years from age 25 to age 65, payable at the moment of death. But she only wants to pay annual premiums for 20 years. What is the amount of the annual premium? For this is example use the Mxm(age, mth, person) function with mth set to ∞. For life insurance, mth determines when the benefit is paid. A positive number means that it is paid at the end of the period of death. A negative number means that it is paid at the beginning of the period of death. For example, if mth = 12, the death benefit is paid at the end of the month of death. If mth = 1, it is paid at the end of the year of death. And if mth = ∞, it is assumed to be paid at the moment of death. If you noticed that the sign of mth means the reverse for life annuities, you are correct. Again, this is a consequence of the mathematical definition of the actuarial functions.

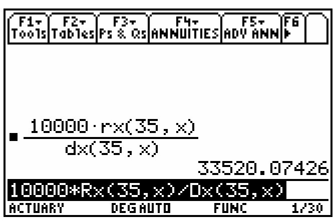




The annual premium is $1,230.80.

Example 18: Increasing Insurance

What is the APV of a life insurance policy that pays $10,000 the first year if death occurs and increases by $10,000 each year after to X who is age 35?



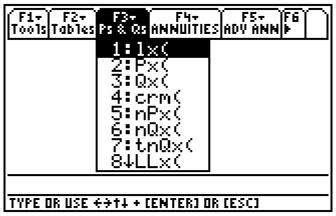
The APV is $33,510.07.

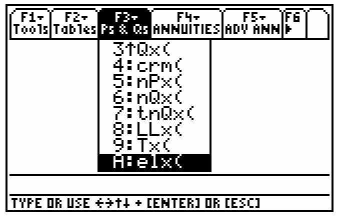
6. Non-integral Ages and the Uniform Distribution of Deaths

All the commutation functions accept non-integral ages, e.g. 50.5 or 32.3. Their values are calculated on the assumption that deaths during the year follow a uniform distribution (the variable Extrapol() is set to udd). However, if you want a straight liner interpolation between non-integral ages, set Extrapol() to lin. If non-integral ages are used with function such as Nxm() or Mxm() where abs(mth) > 1 and assuming udd, there are small relative errors in the calculations. Towards the end of the mortality table the errors can become large. If non-integral ages are used only when abs(mth) = 1 or if only integral ages are used when abs(mth) > 1, there is no error given the uniform distribution of deaths assumption.

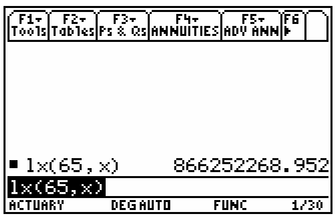
7. Q's and P's

The Q's and P's associated with actuarial math are the probabilities of dying and living. (In actuarial textbooks they are always lower case. I use upper case because the TI-89 does not do subscripts.) The Q's and P's menu is under F3.

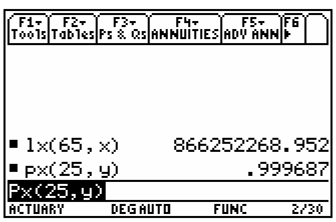




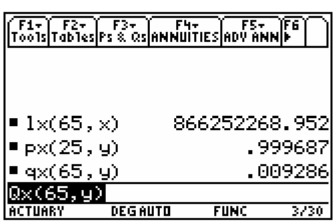
lx(age, person) returns the number of persons living at the beginning of age x for person X, Y or J. The ages extend from age α to age ω, the beginning and ending ages of the current mortality table. The number of persons at age α is the number chosen for the radix of the mortality table, in this case 1,000,000,000. For instance, the number of persons X living at the beginning of age 65 is 866,252,268.952.



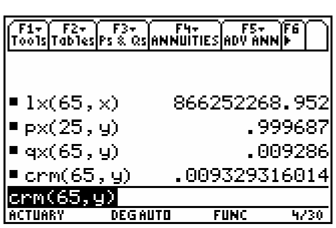
Px(age, person) gives the probability of a person of age x living one year. For Y age 25 the probability of living one year is 0.999687. Px() = l_{x+1}/l_x



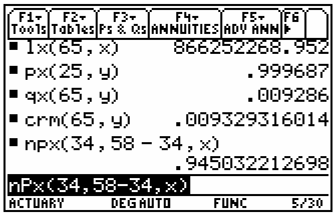
Qx(age, person) gives the probability of a person of age x dying in the next year. For Y age 65 the probability of not being alive at age 66 is 0.009286. Qx() = $(l_x - l_{x+1})/l_x$



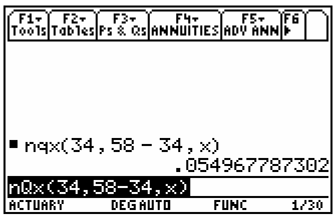
crm(age, person) returns the central rate of mortality of a person of age x. For Y age 65 the central rate of mortality is 0.009329316014. crm() = $(l_x - l_{x+1})$/LLx(age, person) where LLx() is defined below.



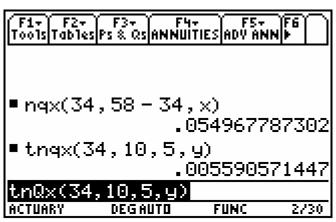
nPx(age, n, person) gives the probability of a person of age x living n years. For X age 34 the probability of reaching age 58 is 0.945032212698. nPx() = l_{x+n}/l_x



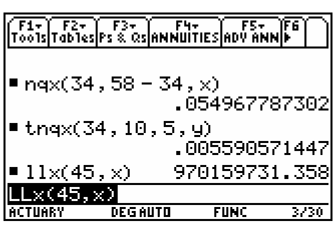
nQx(age, n, person) of course is the probability of a person of age x dying before reaching age x+n. For X age 34 the probability of dying before reaching age 58 is 0.054967787302.
nQx() = $(l_x - l_{x+n})/l_x$



tnQx(age, t, n, person) returns the probability of a person of age x living t years and then dying in the next n years. For Y age 34 the probability of living 10 years and they dying in the next 5 years is 0.005590571447. tnQx(age, t, n person) = nPx(age, t, person) * nQx(age+t, n, person)

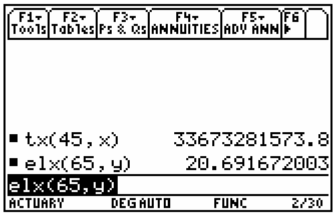


LLx(age, person) gives the number of life-years lived by the l_x persons who attain age x over the year from age x to x+1. On the assumption that deaths are uniformly distributed,
LLx() = $(l_x + l_{x+1})/2$. For example, the number of life-years lived over the year by the X persons who attained age 45 is 970,159,731.358 years.



Tx(age, person) returns the total future lifetime of the l_x persons who attain age x. The total future lifetime of the X persons who attained age 45 is 3,367,321,573.8 years.
Tx() = $l_x/2 + l_{x+1} + \ldots + l_\omega$

elx(age, person) gives the complete expectation of life of a person age x. For Y age 65 the expected life is 20.69 years. elx() = Tx()/lx()



8. Conclusion

I hope you've enjoyed this brief excursion into actuarial mathematics. This is only a taste of what the field has to offer. If you have any questions, comments, suggestions please email me at don.phillips@gmail.com.