Interpolation using the TI-84 Plus CE Graphing Calculator

Version 2.1

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Important Information

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Contents

What's New

What's New in INTRPOL8 v2.1

The midpoint can now be calculated for two points.

You can now save the data you manually enter temporarily to any of the lists L_1 through L_6.

When you load data from a list INTRPOL8 now shows you the data it loaded. Also, INTRPOL8 shows the list name in the main menu heading and in various other screens so you don't have to remember which list you loaded. If you manually entered data INTRPOL8 denotes it as “**`Ma`**”.

The maximum amount of data that can be used for EVERETTs formula has been increased from 8 to 10 data points. Additionally, BESSELs formula has been added and can be used instead of EVERETTs formula.

Previously, the STIRLINGs formula allowed either 3 or 5 data points. Now, STIRLINGs formula has been expanded to allow you to enter 7 and 9 data points.

A DATA VIEW SCREEN has been added to show all your data prior to analysis.

The result screen has been changed to show all your data along with the results.

Introduction

If you have a function you can use the CALC menu to find an unknown value, find a zero of that function or find a maximum or a minimum. But what if you don't have the function? Instead you only have a few equally spaced data points available. It is still possible to obtain an unknown value, find the zero of the function or find a maximum or a minimum using only the available data.

The process of finding unknown values that are intermediate to the known data is referred to as *Interpolation* and it is based on the *Calculus of Finite Differences.* It is a very powerful method of data analysis when you only have a few data points. In general, the data is in the form of a table, however, it is not required that it come from a textbook. It can also have been generated by your calculator or by an unknown function. You can even obtain the data from a laboratory experiment. In these cases using only a few data points you can obtain a result that is fairly close to the answer you would have obtained if you knew the original function the data was based on.

The tabular data will be in a form similar to this.

$$\begin{array}{lll} x_{-2} & f_{-2} & \\ x_{-1} & f_{-1} & \\ x_{0} & f_{0} & \left.\begin{matrix} \\ \\ \end{matrix}\right] f_n \\ x_{+1} & f_{+1} & \\ x_{+2} & f_{+2} & \\ x_{+3} & f_{+3} & \end{array}$$

Where (x_0, f_0) is the central value. The tabular arguments in column x are uniformly separated. Often, these arguments are given in units of time, such as daily or seconds but they can be given in other units, such as meters, radians or simply n or x. In order to obtain an unknown element f_n for any instance which is usually between f_0 and f_{+1} (but in some special cases can be between f_0 and f_{-1}) you will make use of interpolation.

There are many methods that can be used to interpolate for the unknown f_n. The method used depends on the data in column f. For instance, if you have data that is increasing or decreasing uniformly you would use *linear* interpolation. However, if the data reaches a maximum or minimum you would use a different form of interpolation called *extremum*. If the data reaches a value of zero at some point then you would use *zero of a function*. And there are several non-linear methods that can be used when there is some curvature in your data. Another type will give the exact mid-point of the data $f_{½}$ between f_0 and the following value f_{+1}. There is no one type that works for every situation so you must carefully choose one that will work.

In addition, you can interpolate using different numbers of data points of f. The three main methods of interpolation in this app all allow either 3 or 5 data points. But there are some that only need two data points and some that can use more than 5 up to a maximum of ten.

For all interpolations you will need to carefully select the tabular data that you will be using because this will effect the accuracy of the result you obtain. This data should be chosen in such a way that the unknown f_n lies between the central value f_0 and the following value f_{+1}. You should also decide at this time how many data points you want to use. In general, the more data you use the more accurate the interpolated result will be.

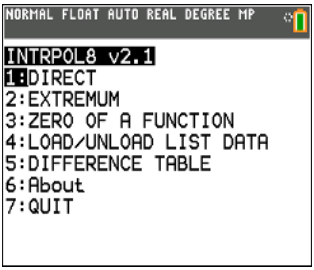
INTERPOLATION FACTOR

The interpolation factor, denoted here by N, is the distance that the unknown value f_n is from the central value f_0 and it is needed only for some interpolation types. It is calculated using the corresponding arguments x_o, x and x_{+1}.

$$N = \frac{(x - x_0)}{(x_{+1} - x_0)}$$

The value of x is the argument which corresponds to the unknown you are looking for (see the 2-POINT FORMULA example on pages 4). The value of N is positive if $x > x_0$ and negative if $x < x_0$. If it is correctly chosen N will be between –½ and +½, but this app will usually give correct results even if N is between –1.0 and +1.0.

INTRPOL8



The program INTRPOL8 is able to give you an interpolated result for several types of tabular data. These types are 'DIRECT', 'EXREMUM', and 'ZERO OF A FUNCTION'. You also have the options in the MAIN MENU to 'LOAD/UNLOAD LIST DATA', create a DIFFERENCE TABLE, check out who is taking credit for this program and to QUIT. QUIT will return you to the TI-84 Plus CE home screen and restore the registers

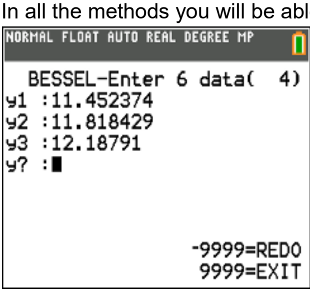
Determine the type of interpolation needed and choose the appropriate method or if you have already entered data into one of the six lists L_1 through L_6, you can load that data into INTRPOL8.

DATA ENTRY

You have two options as far as data entry goes. The first is to enter your data manually using the DATA ENTRY screen, which is described next. The second option is for when you have already entered data into one of the six LIST variables L_1 through L_6 prior to running INTRPOL8. This can be done either using braces and STO on the home screen or using the STAT LIST editor. In either case INTRPOL8 will get the data from the list, starting at location (1) and use it with the method you select. INTRPOL8 also allows you to store manually entered data into one of the lists. However, this data will be deleted when you QUIT INTRPOL8.

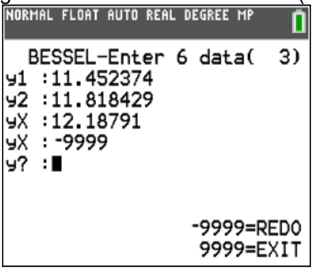
MANUAL DATA ENTRY

In all the methods you will be able to enter data using the DATA ENTRY screen. If



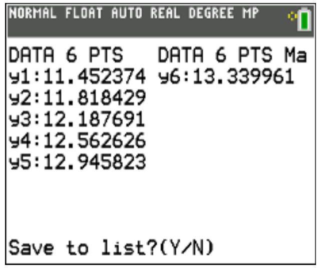
data has not already been loaded using the LOAD/UNLOAD menu you will automatically be taken to this screen when you select a method. Here you will be able to enter your data as shown in the screen to the left. You will also be able to make corrections by entering the number –9999 (REDO) and QUIT by entering 9999 (EXIT) before you have finished entering all of your data.

Examine the screen to the left. It tells you that you are entering the 6 data points BESSEL method. The (4) tells you that you are currently entering the 4th data point. You will enter data where y? appears. After you have entered the data the ? will be replaced by the number that appears inside the ( ). As shown in the next image to go back and make a correction (REDO) simply enter the number -9999. When you

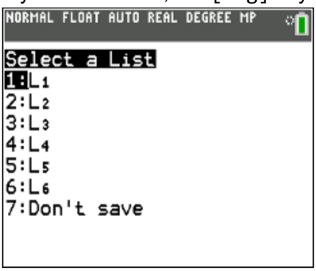


have done this note that the current entry is decreased to the previous entry (3). Additionally, an X replaces the entry number next to y to let you know that that data is now invalid and will not be saved.

At any point during your data entry you can quit entering data and return to the previous menu without saving the data by entering 9999.



As shown to the right after you have entered all of your data it will be re-displayed on the DATA VIEW SCREEN. Note that the errors made above have been removed. The **Ma at** the upper right signifies that this data was manually entered. You will be given the option to save this data to one of the six lists. Press 'Y', the `[1]` key for YES or 'N', the `[log]` key for NO.



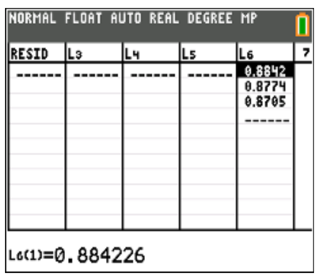
If you select 'Y' you will be taken to the SELECT A LIST menu shown to the left. Select one of the six lists and your data will be stored in that list for the duration of this app's execution starting at list position (1) and overwriting any data that may already be stored there. Afterwards you will return to the method you selected.

You can continue with the method without saving to a list by selecting 'Don't save'.

DATA ENTRY USING A LIST - (LOAD/UNLOAD LIST DATA)

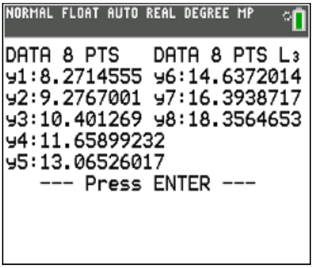
If you prefer you can enter the data points using one or more of the six LIST variable L_1 through L_6 prior to running INTRPOL8 in a manner similar to this.

$\{0.884226,0.877366,0.870531\}\rightarrow L_6$



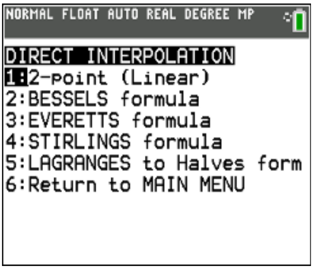
or you can enter the data directly using the stat list editor as shown to the right.

To use a LIST you will load it from the MAIN MENU of INTRPOL8. Simply select the 'LOAD/UNLOAD LIST DATA' option where you will be taken to the SELECT A LIST menu, similar to the one shown above. Here you select the LIST you want the method to load. INTRPOL8 will automatically retrieve that data starting at location (1) and then display the data in the DATA VIEW SCREEN (below). The upper right corner displays the ID of the list that was loaded (L_3 in this case). Press ENTER to return to the MAIN MENU.



When data is LOADED the MAIN MENU banner will change to 'INTRPOL8 v2.1-L_x data;n'. Where n is the amount of data loaded from the list. If, after loading data, you decide you want to manually enter data or use another list you can UNLOAD the current data just by selecting 'LOAD/UNLOAD LIST DATA' again. Only the last list loaded will be used. The list data must always start in location 1 and a maximum of 10 data points will be loaded. That is, for L_1 the data loaded will be $L_1(1)$ to $L_1(10)$. If you try to use a list that is empty you will receive a NO DATA IN THAT LIST error message.

DIRECT



DIRECT interpolation is used with data that is uniformly increasing or decreasing. In addition to using 2-point (Linear), you can select BESSELS, EVERETS, STIRLINGS or LAGRANGES TO HALVES formulas. The last item allows you to return to the MAIN MENU.

EVERETTS formula allows you to enter from 2 to 10 data points. Increasing the amount of data will, in general, increase the accuracy of a result.

The BESSELS formula allows you to enter 2, 4, 6, 8 or 10 data points and STIRLINGS formula allows you to use 3. 5. 7 or 9 data points.

The LAGRANGES to Halves formula will give a result that is exactly halfway between f_0 and f_{+1}. This is useful for data that has some curvature.

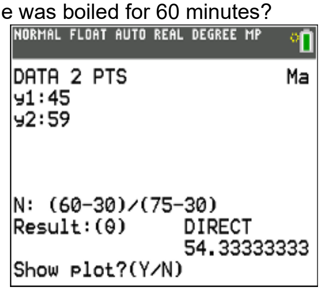
2-POINT FORMULA

The 2-point formula is the simplest of all methods and is just a linear (straight line) method. You would use it to find an unknown on a straight line.

In two experiments on the determination of tin, some soil was split into two samples and digested by boiling using hydrochloric acid for different lengths of time. The results of the analysis are as follows:

Refluxing time	Amount of tin found
30 min	45 mg/kg
75 min	59 mg/kg

How much tin would you expect to recover if a sample was boiled for 60 minutes?



1. The interpolation factor is

$$N=\frac{60-30}{75-30}=0.67$$

2. Select [1] to select DIRECT and [1] to select 2-point (Linear). You will be taken to the DATA ENTRY screen. Enter the data as follows:

y1 : 45
y2 : 59

3. Press 'N' ([log] key) to not save this data to a list. Enter the interpolation factor:

N:(60–30)/(75–30)

4. The result is displayed: Result:(θ) DIRECT

54.33333333

Show plot?(Y/N)

5. For now select 'N' to not show the plot.

See the SHOW PLOT section for an explanation of this option. You would recover 54 mg/kg after 60 minutes. The (θ) denotes that the RESULT is also stored in the θ variable. You can access it when you QUIT INTRPOL8. Above the result is the type of interpolation that was performed i.e. DIRECT.

STIRLINGS FORMULA

STIRLINGS is more exact than other interpolation methods and works best when N, the interpolation factor, is between -¼ and +¼ but it works well for -½ ≤ N ≤ +½. STIRLINGS allows you to enter 3, 5, 7 or 9 data points For the example we will choose to use the 5-point method. The 3, 7 and 9-point methods all work in a similar way.

5-POINT FORMULA

FM radio signals can only travel in a straight line from the broadcast antenna to the horizon. Therefore, the area where you can pick up any FM signal on your car's radio must be within the visible horizon of that station's antenna. So the taller the antenna the further the horizon will be and therefore the further the FM signal will reach. If your favorite FM station is using an antenna that is 319 m high use the following table to figure out how far away you can travel in your car and still receive that FM station.

Distance of visible horizon for given heights above earth's surface					
height (m)	100	200	300	400	500
distance to horizon (km)	35.7	50,5	61.9	71.4	79.9

1. The interpolation factor is $N = \frac{(319-300)}{400-300} = 0.19$ Since the interpolation factor is between -¼ and +¼ (i.e -0.25 and +0.25) it is appropriate to use STIRLINGS.

2. Select [1] to select DIRECT and [4] to select STIRLINGS formula then select [2] to pick 5-point formula.

3. You will be taken to the DATA ENTRY screen.. When prompted enter these distances:

y1 : 35.7
y2 : 50.5
y3 : 61.9
y4 : 71.4
y5: 79.9
N : 0.19

4. The result is displayed: RESULT:(θ) STIRLINGS

63.81544672.

The result has been stored in the θ variable. You should be able to listen to your favorite FM station up to 63.8 km (39,2 miles) away.

BESSELS FORMULA

If N, the interpolation factor, is between +¼ and +¾ use BESSELS or EVERETTS FORMULA. BESSELS allows you to enter 2, 4, 6, 8 or 10 data points For the example we will choose to use the 6-point method. The 2, 4. 8 and 10-point methods work the same.

6-POINT FORMULA

The Apollo 11 Lunar Lander touched down on the surface of the Moon on 1969, July 20 at 20:18 GMT. Using data obtained from the *Astronomical Ephemeris* interpolate the Moon's position in the sky at that moment.

Moon's coordinates for 0^h and 12^h GMT for July 1969

	1969		α	δ	
x_{-2} =	July 19.5	f_{-2}	= $11^h.452374$	$+3°.688044$	
x_{-1} =	July 20.0	f_{-1}	= $11^h.818429$	$+0°.717666$	
x_0 =	July 20.5	f_0	= $12^h.187691$	$-2°.289686$	1969, July 20, 20:18 GMT
x_{+1} =	July 21.0	f_{+1}	= $12^h.562626$	$-5°.308878$	
x_{+2} =	July 21.5	f_{+2}	= $12^h.945823$	$-8°.312494$	
x_{+3} =	July 22.0	f_{+3}	= $13^h.339961$	$-11°.269967$	

Interpolation factor

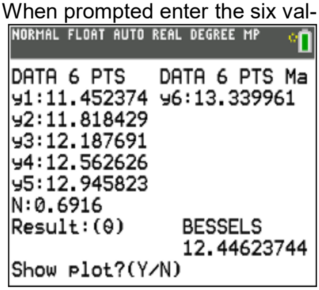
The interpolation factor is measured from the central point x_0 (1969,July 20.5).

1. Find the interval between the data (x_{+1} - x_0) = 21.0 - 20.5 = 0.5 days.
2. Convert the time of the landing into days: $20^h + 18^m/60 = 20^h.30$
   $20^h.30/(24^h/1 \text{ day}) = 0.8458$ day.
   Therefore the landing (x_n) was on July 20.8458.
3. The interpolation interval (N) is found to be: $\dfrac{20.8458 - 20.5}{21.0 - 20.5} = 0.6916$
4. Since N is between +¼ (0.25) and +¾ (0.75) you can use BESSELS FORMULA.

Data entry

5. Select [1] for DIRECT, [2] to select BESSELS and [3] to select 6-point formula.
6. You will be taken to the DATA ENTRY screen. When prompted enter the six values given in column α above.

y1 : 11.452374
y2 : 11.818429
y3 : 12.187691
y4 : 12.562626
y5 : 12.945823
y6 : 13.339961
N : 0.6916



.The result is displayed:

RESULT:(θ) BESSELS
12.44623744 ►DMS $12^h\ 26^m\ 46.455^s$

Repeat these steps entering the values of δ gives the following result:

RESULT:(θ) BESSELS
-4.378091044 ►DMS -4° 22' 41.13"

Comparison with calculated data

Compare these results with data from a computer program which uses the entire lunar theory *Improved Lunar Ephemeris* (j=0), which is the same theory the *Astronomical Ephemeris* and NASA used. It gives these results for this time and date:

α $12^h26^m46^s.455$ δ -4°22' 41″.13

Compare these results to the following results which you would have obtained interpolating using the 4 POINT FORMULA with data points (y2,y3,y4, y5) instead

α $12^h26^m46^s.447$ δ -4°22' 41″.32

This demonstrates that the accuracy of the interpolated result will often depend on the number of data points you use.

EVERETTS FORMULA:2-10 PTS

Everett's formula is a good general purpose method. It works best when the interpolation factor is between +¼ and +¾.

Find the amount of compound interest on a loan at 5⅓% for a period of 48 months. The following equally spaced tabular data is taken from *Financial table of Compound Interest*. The correct rate for 48 months at 5⅓% is 12.11043487.

#	%	Rate
1	4.25	7.37115802
2	4.50	8.27145557
3	4.75	9.27670014
4	5.00	10.40126965
5	5.25	11.65899232
6	5.50	13.06526017
7	5.75	14.63720146
8	6.00	16.39387173
9	6.25	18.35646534
10	6.50	20.54854961

1. Select [1] to select DIRECT and [3] to select EVERETTS formula
2. Number of points: 2
3. You will be taken to the DATA ENTRY screen.. When prompted enter these two values:

y1 : 11.65899232
←f_0 y2 : 13.06526017
N : (5.33–5.25)/(5.50–5.25)

4. The result is displayed:

RESULT:(θ) EVERETTS
12.10899803

With only two points we obtained 3 digits of accuracy.

Now, repeat using 3 data points and N; = 1/3

y1 : 10.40126965
y2 : 11.65899232
y3 : 13.06526017
RESULT:(θ) 12.11857881

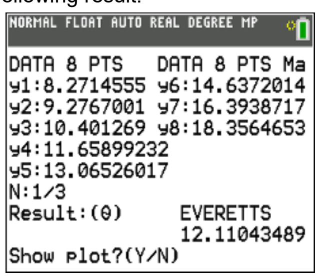
With three data points we gained an additional digit of accuracy.

Repeating using 4 data points and N: 1/3 yields

y1 : 10.40126965
y2 : 11.65899232
y3 : 13.06526017
y4 : 14.63720146
RESULT:(θ) 12.11039741

Using 8 data points would have gotten you the following result:

y1 : 8.27145557
y2 : 9.27670014
y3 : 10.40126965
y4 : 11.65899232
y5 : 13.06526017
y6 : 14.63720146
y7 : 16.39387173
y8 : 18.35646534
N: 1/3
RESULT:(θ) 12.11043489



Using all ten data points gives the same result as eight data points:

RESULT:(θ)   12.11043489

LAGRANGES TO HALVES

TO HALVES returns the exact mid-point of 2, 4 or 6 equally spaced tabular values. This method can be used for instances when your data has some curvature.

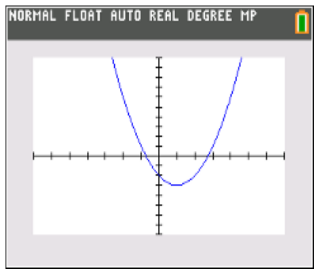
1. Select [1] for DIRECT and [5] to select LAGRANGES to Halves formula.
2. Next select [2] 4-POINT FORMULA.
3. When prompted in the DATA ENTRY screen enter the following:

y1 :1128.732
y2 :1402.835
y3 :1677.247
y4 :1951.983

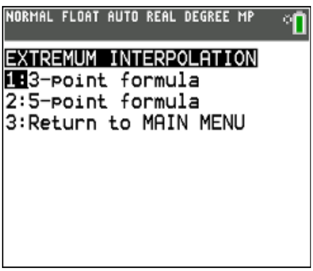
3. The result is displayed: is exactly HALF-WAY:(θ)
1540.001438

EXTREMUM

An extremum is reached when the data arrives at a maximum or a minimum. In the image to the right the curve reaches a minimum at (1,-3).



You can use interpolation to find the maximum or minimum in a set of data without having the equation that the data is based on available to you.



The EXTREMUM interpolation method allows you to find the extremum from either 3 or 5 data points. You get to the EXTREMUM menu by selecting [2] in the MAIN MENU. You can always return to the MAIN MENU by selecting [3] from the EXTREMUM menu.

3-POINT FORMULA

From the curve shown above the following set of data was obtained.

x	*f*
-1.5	+3.25
-0.5	-0.75
+0.5	-2.75
+1.5	-2.75
+2.5	-0.75

Find the minimum value of that curve using only three data points.

The first thing you must do is examine the data and decide which three points should be used. Chose the three points that are on either side of the minimum, which on the graph appears to be around x = +1.0. To find the minimum complete the following steps.

1. Select [2] to select EXTREMUM and [1] to select 3-point formula.
2. When prompted enter the following 3 data points:

y1 : -0.75
y2 : -2.75
y3 : -2.75

3. The result is displayed: minimum Y is:(θ) -3
Interpolation Factor (N): 0.5

These results are telling you the following.

a) The MINIMUM Y is at -3.0. This result is stored in θ. Examining the graph in the above image you can see that the minimum does in fact appear to be at -3.0.

b) The second result is the Interpolation Factor N. You use it to get the corresponding value of X as follows. Note that the Interpolation Factor is stored in N.

i.) Select [1] DIRECT and because N ≤ +½.pick [4] STIRLINGS and [1] to use the 3-point formula.

ii.) Enter the three values of X that correspond to the three values of f entered above and enter 'N' when asked for the interpolation factor:

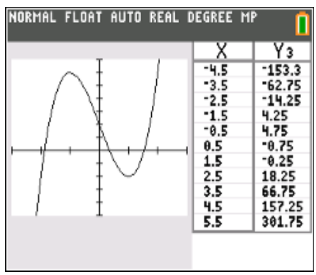
y1 : -0.50
y2 : +0.5
y3 : +1.5
N : N

iii.) The result is displayed: RESULT:(θ) STIRLINGS
1

The minimum X value is +1.0. So the minimum is found to be (+1.0, -3.0).

5-POINT FORMULA

Find the maximum and the minimum of the graph below using only the data given. Since you don't have the equation of this function you can't use the CALC menu. You can use EXTREMUM interpolation using 5 TABULAR POINTS to come up with the answers.



#	X	Y
1	-3.5	-62.75
2	-2.5	-14.25
3	-1.5	+4.25
4	-0.5	+4.75
5	+0.5	-0.75
6	+1.5	-0.25
7	+2.5	+18.25

Begin by looking over the data and carefully selecting the data you will use to perform the interpolations. Since you are going to use 5-point interpolation you need to select 3 points that are on one side of each extremum and 2 points that are on the other side,

The data given in the table to the left was taken from above. Notice that three of the data points will be used twice (#3, #4 and #5). Once to find the maximum and again to find the minimum.

Store the following data points in the list variables L_1 and L_2.

$\{-62.75,-14.25,4.25,4.75,-0.75\} \rightarrow L_1$
$\{4.25,4.75,-0.75,-0.25,18.25\} \rightarrow L_2$

1. Select [4] LOAD/UNLOAD LIST DATA than select [1] for L_1 and then [enter]. The banner at the top changes to INTRPOL8 v2.1- L_1 data:5 The 5 tells you that it loaded 5 data points from list L_1.
2. Next select [2] EXTREMUM followed by [2] 5-POINT FORMULA The BUSY INDICATOR [*] will appear briefly.
3. The result is displayed:

maximum Y reached at:(θ) 6
Interpolation Factor (N):
0.4999999

Checking the graph from the display shown above the maximum at Y = +6 appears to be correct.

4. Press [N] then select [4] LOAD/UNLOAD LIST DATA to unload the L_1. data.
5. Select [1] DIRECT. [4] STIRLINGS and [2] 5-point formula.

Enter the corresponding values of X (#1-#5) and N to get the result for X: -1.0. So the maximum is found to be at (-1,+6)

6. Then [4] LOAD/UNLOAD LIST DATA followed by [2] for L_2.
7. Select [2] EXTREMUM and [2] 5-point formula. The [*] appears again for quite a long while.
8. The result is displayed:

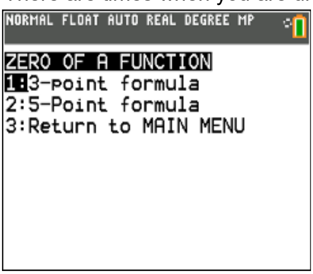
minimum Y is:(θ) -1.990624
Interpolation Factor (N):
0.5392742

Using the result for N and STIRLINGS formula with the corresponding x values yields theresult for X of 1.039274222. So the minimum is at (1,-2) Checking the graph, that appears to be correct.

The function that defines this graph is: $2X^3$-6X+2. Now if you use the CALC menu item 'maximum' on this function you should get X = -1, Y= 6, and then using the 'minimum' menu item will give X= 0.9999973 and Y = -2. These results match the maximum and minimum results INTRPOL8 gives using only 5 data points.

ZERO OF A FUNCTION

There are times when you are unable to calculate the interpolation factor N directly.



In these cases you need to find an alternate means to obtain the interpolation factor so you can use it to find the interpolated result. You can use ZERO OF AFUNCTION to determine what value of X will give Y = 0.

As with the other methods you can select either 3-POINT FORMULA and 5-POINT FORMULA and you can use data loaded from a list. You can also Return to MAIN MENU.

3-POINT FORMULA

In the following example you can not interpolate the result directly because you can't calculate the interpolation factor N. In this case you will need to find the interpolation factor, which you can then use to find the answer.

You performed the following experiment. You placed 100 mL of 78°F room temperature water into several cups and heated them in a microwave for various lengths of time then you measured their temperature. From the data given you must deduce the amount of time it would take for the water to reach 100 °F.

Time in microwave	Temperature (°F)
0 sec	78 °F
10 sec	92 °F
20 sec	113 °F

1. Select [3] ZERO OF A FUNCTION then [1] 3-point formula. If you let 100 be your zero then your y values are:

y1: 78 – 100 = -22 °F
y2: 92 – 100 = -8 °F
y3:113 - 100 = +13 °F

2. The result is displayed: Y IS 0 AT N= (N)
0.421594476

N is your interpolation factor and it has been stored in the N variable.

You can now use this to get the time. Since N is between +¼ and +¾ you can use BESSELS or EVERETTS formula. Since you have 3 data points and BESSELS only uses even numbers of data that leaves you with EVERETTS formula.

3. Select [1] DIRECT. [3] EVERETTS and NUMBER OF POINTS: 3. This time enter the times instead of the temperatures and enter the N variable for the interpolation factor::

y1: 0
y2: 10
y3: 20
N : N

4. The result is displayed: RESULT:(θ) EVERETTS
14.21594476

It should take 14.2 seconds in the microwave to heat up 100 mL of water to 100°F. So next we try this out ourselves by microwaving 100 mL of water for 14 seconds. The temperature we got was 99°F.

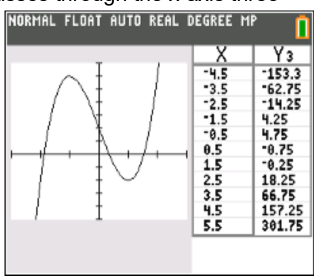
5 POINT FORMULA

Returning to the data we examined in the 5-point extremum example, this time we will find when the function passed through the x-intercept.

As can be seen in the graph to the right the line passes through the x-axis three times.

#	X	Y
1	-4.5	-153.3
2	-3.5	-62.75
3	-2.5	-14.25
4	-1.5	+4.25
5	-0.5	+4.75

1. The table to the right lists the first 5-points in the table shown with the graph. These five points surround the far left of the three intercepts.
2. Enter the five values in the Y column.
3. The result is displayed.



Y IS 0 AT N= (N)
0.6206869317

Using this result for N and STIRLINGS formula with the five X values yields this result for X: -1.879313068. So the first x-intercept is at -1.8793.

It is left up to you to find the other two x-intercepts. You should get the following results for N: 0.8472963551 and +0.0320888865, which gives these results:

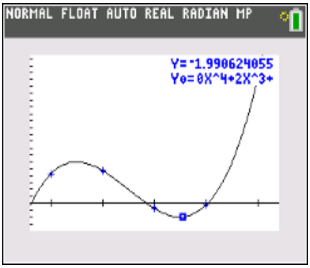
x-intercept #2: +0.34730
x-intercept #3: +1.5321

Again, using the CALC menu with the function $2X^3-6X+2$ and selecting 'zero' will result in the following three answers:

x = -1.8794
x = +0.34730
x = +1.5321

SHOW PLOT

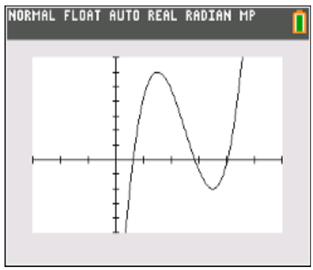
An option you are given with most interpolation methods is to show the data on a graph plot. Along with showing the data the interpolated result obtained by the method you selected is also plotted on the graph.



The plot to the left is what you should get using the L_2 list given in the example for the 5-point extremum method. The + signifies the 5 data points obtained from the L_2 list. The □ is the result from the interpolation. The result of the interpolation is also shown as Y= and the function that was created is shown as Y_0=.

FUNCTION GENERATION

SHOW PLOT will also calculate a function that fits the data and then uses that function to display a solid line on the above graph. If you QUIT INTRPOL8 and use function graphing to display the Y_0 function you will get a plot similar to the screen shot to the right. It resembles the plot on page 11 only ofset along the x-axis. Keep in mind that this was done using only five data points.

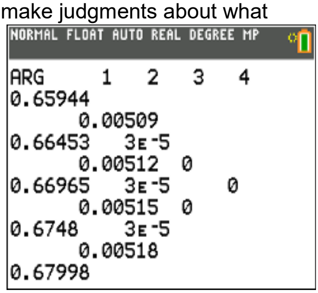


The function is determined by the number of data points you provided. It uses one of the various regression methods available in the STAT CALC menu: LinReg, QuadReg, CubicReg or QuartReg. The function that was created is stored in Y_0 where you can then use it.

DIFFERENCE TABLE

A difference table is a common way that data is displayed prior to interpolation. By examining a difference table of your data you can make judgments about what methods you can n use for interpolation.

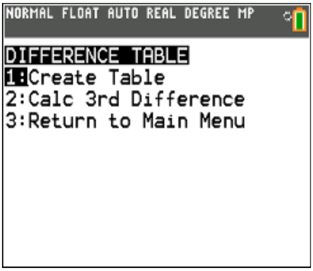
Argument	1st	2nd	3rd	4th
0.65944				
	+509			
0.66453		+3		
	+512		0	
0.66965		+3		0
	+515		0	
0.67480		+3		
	+518			
0.67998				



The above table is a common way that a difference table would be shown in a text-book and to the right is the same data using INTRPOL8.

The column labeled 1st is the first difference of the elements, 2nd is the second difference and columns 3rd and 4th are the third and fourth differences.

For most methods you are provided with at least two options for interpolating: 3-point formula and 5-point formula. By examining the difference table of your data you can decide if you can simply use the three point method or if it would be better to use the five point method. As a general rule when the second differences are almost constant and the third differences are almost zero than it is appropriate to use the three point method. Otherwise you should use the five point method.



You are given a couple of options when you select DIFFERENCE TABLE. You can create a difference table using CREATE TABLE or you can calculate the third difference using four data points. For both menu items if you have loaded data using LOAD/UNLOAD LIST DATA then that data will automatically be used. However, if you have not loaded any list data then both of these menu items will let you enter data manually.

CALC 3RD DIFFERENCE

This item will calculate the third difference for you so you don't have to examine a difference table. You do need 4 data points to use this procedure.

Using the data shown in the difference table above find the third difference.

$$\{0.65944, 0.66453, 0.66965, 0.67480, 0.67998\} \to L_6$$

1. Select [4] LOAD/UNLOAD LIST DATA then select [6] for L_6.
2. Next select [5] DIFFERENCE TABLE followed by [2] CALC 3RD DIFFERENCE
3. The difference is displayed:

3rd difference is:

0

Which is the result you would expect examining the top item in the 3rd column in the difference table shown above. This means you can use only 3 data points and expect to obtain a very accurate result.

BUSY INDICATOR

Some of the methods use a process called iteration to obtain an answer. This process sometimes takes several seconds or a minute. While this is occurring INTRPOL8 displays an animated BUSY INDICATOR [∗] to let you know that it is busy with a calculation. Be patient. INTRPOL8 will kick out of the iteration if no solution is found, that is, it will not get stuck in an infinite loop.

PRESERVING MEMORY

INTRPOL8 automatically preserves the contents of registers A through Z and the list registers L_1 through L_6 before running and then restores these registers when the program exits through QUIT. In this way you can use INTRPOL8 when you have data stored and not have to worry that your data will be lost.

This is done by running the subroutine called θBACKUP. This subroutine makes a backup copy of the registers A through Z and L_1 through L_6 in the user defined list called $_L$BKUP. It then restores these registers when you QUIT.

After manually entering data you will be asked if you would like to save that data to one of the six list registers. That data will only stay in the list register until you QUIT INTRPOL8. When you QUIT the original data that was in those six list registers will be restored via the data stored in $_L$BKUP.

ERROR MESSAGES

INTRPOL8 has rudimentary error handling. Below is a list of errors that may occur.

ERROR - INVALID DATA - The data gave a result that is equal to zero or the result would cause an overflow so INTRPOL8 stops before letting that occur.

INVALID DIMENSION - May occur if you loaded data from a list which contained less data then the method requires. That is, if the list only contains 3 data points and you asked to interpolate using 5 data points.

INVALID N - This error will only occur when the method asks for the interpolation factor N and you enter the value -9999.

NO DATA IN THAT LIST - If you try to use a list that is empty you will receive this error

NOT ENOUGH DATA - If you load data from a list that does not contain enough data for the method you selected you will get this message.

UNDEFINED variable used is not currently defined – This error occurs if you have deleted one or more of the list variables L_1 - L_6 . This occurs because the first thing INTRPOL8 does is make a backup copy of L_1 to L_6 and A-Z. To fix this manually redefine the missing list.

ERROR: DIMENSION MISMATCH – This error may occur when you press the [graph] button after quitting INTRPOL8. Simply press stat plot [2nd][Y=] and turn Plot1 Off.

BUG REPORTS

Report any bugs to me at the email address below. Be sure to give an example that will reproduce the bug along with the version of INTRPOL8 you have.

email: sthomasbradley@gmail.com

PRINTING THIS BOOKLET

This booklet was designed to be printed out as a small 4¼ by 5½, easy to carry booklet. To do this simply follow these steps.

1. In Adobe Reader select File > Print,
2. In Page Sizing and Handling > Multiple
3. Pages per sheet: 2
4. Orientation: Portrait
5. Page order: Horizontal
6. Print pages:20,1,18,3,16,5.14,7,12,9
7. Print
8. Without shuffling the pages, flip them over and put them back in the paper tray.
9. Without changing any settings print the back pages in this order: 2,19,4,17,6,15,8,13,10,11

10. Now take the pages out of the printer and fold the pages in half so that the title page appears on top after folding.
11. Staple or glue these pages together into a booklet.