Motion3d() 3 Dimensional Motion with Time Dependent Acceleration 2-25-08

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This program analyzes 1, 2, or 3 dimensional motion with either time dependent or constant acceleration. The values of any 10 of the 16 quantities { x_o, y_o, z_o, v_{ox}, v_{oy}, v_{oz}, a_x, a_y, a_z, x, y, z, v_x, v_y, v_z, t } must be entered, then the program solves for the remaining 6 unknown quantities from the list. Either position, velocity, or acceleration can be entered as a function of time for each of the 3 dimensions. For example, v_x, y, and a_z could all be entered as functions of time. The number of unknowns must always be 6. The initial time is assumed to be t_o=0. Only solutions with t>0 are given. Any angles entered are assumed to be in radians, so if one in degrees is entered, a degree symbol (°) must be used. In time dependent cases, the values of the unknowns as well as expressions for them as functions of time are given. These are all copied to the home screen.

Copyto_h(), by Samuel Stearley, is used to copy the solutions to the home screen so they can be more easily used in further calculations.

For 2 dimensional motion with constant acceleration, see Planemo().
For projectile motion, see Projctle().
For projectiles in a resistive medium, see Projdrag().

Place Motion3d() and Copyto_h() in the same folder, then runMotion3d().

Ex 1. A projectile is launched from (0, 0, 0) with initial velocity $\vec{v}_o = \hat{\mathbf{i}}(20) + \hat{\mathbf{j}}(30) + \hat{\mathbf{k}}(-15)$ m/s.
It lands at (x, 25 m, z).

a) Find the coordinates of the landing point and the time of flight.
b) Find the maximum altitude reached.

a)

Input			
	$x_o = 0$	$y_o = 0$	$z_o = 0$
	$v_{ox} = 20$	$v_{oy} = 30$	$v_{oz} = -15$
	$a_x = 0$	$a_y = -9.81$	$a_z = 0$
	$x = x$	$y = 25$	$z = z$
	$v_x = v_x$	$v_y = v_y$	$v_z = v_z$

Eqs.
$t(t-6.1162) = -5.0968$ $x = 20t$ $z = -15t$ $x = 20t$ $v_x = 20$ $v_y = 30 - 9.81t$ $v_z = -15$

Sol. #1	$t = 5.121$	$x = 102.4$	$z = -76.81$	$v_x = 20$	$v_y = -20.24$	$v_z = -15$
Sol. #2	$t = .9953$	$x = 19.91$	$z = -14.93$	$v_x = 20$	$v_y = 20.24$	$v_z = -15$

b) Edit previous result to find maximum altitude:

Input			
	$x_o = 0$	$y_o = 0$	$z_o = 0$
	$v_{ox} = 20$	$v_{oy} = 30$	$v_{oz} = -15$
	$a_x = 0$	$a_y = -9.81$	$a_z = 0$
	$x = x$	$y = y$	$z = z$
	$v_x = v_x$	$v_y = 0$	$v_z = v_z$

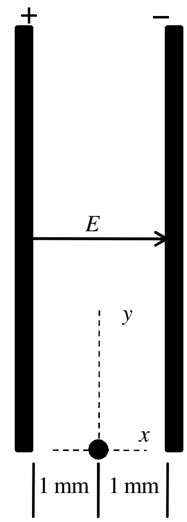
Sol. $t = 3.058$ $x = 61.16$ $y = 45.87$ $z = -45.87$ $v_x = 20$ $v_z = -15$

Ex 2: At t=0, a particle is at position $\vec{r} = \left(10\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right)$ m moving with velocity $\vec{v}_o = \left(20\hat{\mathbf{i}} - 5\hat{\mathbf{j}}\right)$ m/s and acceleration $\vec{a} = \left(\hat{\mathbf{i}}\left(\frac{100}{t+1}\right) + \hat{\mathbf{j}}(2t)\right)$ m/s. Find its position and velocity at t = 20 s.

Input	$x_o = 10$	$y_o = -4$
	$v_{ox} = 20$	$v_{oy} = -5$
	$a_x = \frac{100}{t+1}$	$a_y = 2t$
	$x = x$	$y = y$
	$v_x = v_x$	$v_y = v_y$

Eqs:	$x = 100(t+1)\ln(t+1) - 80t + 10$	$y = .33333t^3 - 5t - 4$
	$v_x = 100\ln(t+1) + 20$	$v_y = t^2 - 5$

Sol : $x = 4803.$ $y = 2563.$ $v_x = 324.5$ $v_y = 395.$



Ex #3. An electron enters a uniform electric field in the region between two charged metal plates. E is is increasing linearly with time so that $\vec{E} = \hat{\mathbf{i}}\left[100 + \left(2\times10^8\right)t\right]$. The electron enters the field midway between the plates, with a velocity of $\vec{v}_o = \hat{\mathbf{i}}\left(-2\times10^4\right) + \hat{\mathbf{j}}\left(3\times10^5\right) + \hat{\mathbf{k}}\left(4\times10^4\right)$ m/s. Determine the coordinates of the point at which the electron hits the positive plate.

Input	$x_o = 0$	$y_o = 0$	$z_o = 0$
	$v_{ox} = -2\times10^4$	$v_{oy} = 3\times10^5$	$v_{oz} = 4\times10^4$
	$a_x = -\frac{qE}{m} = -\frac{\left(1.6\times10^{-19}\right)\left(100 + \left(2\times10^8\right)t\right)}{9.1\times10^{-31}}$	$a_y = 0$	$a_z = 0$
	$x = -.001$	$y = y$	$z = z$
	$v_x = v_x$	$v_y = v_y$	$v_z = v_z$

Eq.	$t\left(t^2 + 1.5\times10^{-6}t + 3.4125\times10^{-15}\right) = 1.70625\times10^{-22}$	$y = 3\times10^5 t$	$z = 4\times10^4$
	$v_x = -1.75824\times10^{19}t^2 - 1.75824\times10^{13}t - 2\times10^4$	$v_y = 3\times10^5$	$v_z = 4\times10^4$

Sol. $t = 9.561\times10^{-9}$ $y = 2.868\times10^{-3}$ $z = 3.825\times10^{-4}$ $v_x = -1.897\times10^5$ $v_y = 3\times10^5$ $v_z = 4\times10^4$

Ex. 4. At t=0, the initial position and velocity of a particle is given by $x_o = 5$ $y_o = 0$ $z_o = 0$ and $\vec{v}_o = \hat{\mathbf{j}}(15) + \hat{\mathbf{k}}(10)$ Its acceleration is $a_x = -45\cos(3t)$ $a_y = -45\sin(3t)$ $a_z = -5t^2$ Find its position and velocity at the instant it reverses its direction of motion along the z-axis.

	$x_o = 5$	$y_o = 0$	$z_o = 0$
	$v_{ox} = 0$	$v_{oy} = 15$	$v_{oz} = 10$
Input	$a_x = -45\cos(3t)$	$a_y = -45\sin(3t)$	$a_z = -5t^2$
	$x = x$	$y = y$	$z = z$
	$v_x = v_x$	$v_y = v_y$	$v_z = 0$

Eq. $t^3 = 6$ $x = 5\cos(3t)$ $y = 5\sin(3t)$ $z = 10t - .41667t^4$ $v_x = -15\sin(3t)$ $v_y = 15\cos(3t)$

Sol. $t = 1.817$ $x = 3.368$ $y = -3.696$ $z = 13.63$ $v_x = 11.09$ $v_y = 10.1$