Routh

v1.02

to **Kaveh**
My “bestest” friend

DISCLAIMER

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Introduction

Routh criterion[1] (Routh, 1905) makes it possible to determine the stability of a system without having to solve for the poles of the system transfer function. Despite the simplicity of its algorithm, when applied to real world control systems, Routh table soon becomes cumbersome to generate by hand.
The purpose of this program is to derive the Routh table, given the characteristic polynomial. Special cases of the showing up of a zero only in the first column as well as the presence of an entire row of zeros are taken care of by the program.

Installation

Upload the file *Routh.89g* to a folder on your handheld. Do not rename any files, or the program will not function properly if it does function at all. You may opt to archive the program files *after* running it for the first time so that TI OS doesn't have to reparse them every time you run the program.

I haven't tested this program on any handheld other than my TI-89 but I suppose it should function with no problem, whatsoever, on TI-89 Titanium and Voyage too.

Execution

Invoke the program with the following syntax

Routh({*coeff*})

where *coeff* represents the coefficients of the polynomial in question. The coefficients should be entered from the highest power present to the zeroth power (constant term). If some power is absent in the polynomial, zero must be entered for coefficient. This, clearly, also applies to the absence of the constant term. Therefore for the following polynomial,

-s^5+2s^4+s^2+s= -s^5+2s^4+0s^3+s^2+s+0

the syntax would be,

Routh({-1,2,0,1,1,0})

Another point worth mentioning is that since the stability of a system involves its closes loop transfer function poles, should you have a feedback representation of a system, you have to derive its *closed loop* transfer function and then feed the appropriate coefficients to the program. Forgetting this point may result in calling the program with the coefficients of the denominator of the *open loop* transfer function of a feedback system and, obviously, getting the wrong results.

[1] Aka Routh-Hurwitz criterion

Interpreting the Routh table

Simply stated, Routh-Hurwitz criterion declares that *the number of roots of the polynomial that are in the right half plane is equal to the number of sign changes in the first column of the Routh table.*

For a system with the following transfer function denominator,

s^3+10s^2+31s+1031

the program yields,

$$\begin{bmatrix} 1 & 31 \\ 10 & 1031 \\ -\frac{721}{10} & 0 \\ 1031 & 0 \end{bmatrix}$$

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which Routh table indicates two sign changes: one from the 2^{nd} to 3^{rd} row and another from the 3^{rd} to 4^{th} row. Hence, there are two RHP poles and the system is clearly unstable.

Special Cases

Two special cases can occur: (1) the Routh table sometimes will have a zero only in the first column of a row, or (2) the Routh table sometimes will have an entire row that consists of zeros.

Zero only in the first column

If the first element of a row is zero, division by zero would be required to form the next row. To avoid this phenomenon, an epsilon is assigned to replace the zero in the first column. The value of epsilon is then allowed to approach zero from either the positive or negative side, after which the signs of the entries in the first column can be determined.[2]

Example. Stability via epsilon method.
For a system with the following characteristic polynomial,

s^5+2s^4+3s^3+6s^2+5s+3

the program yields,

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 3 \\ \varepsilon & 7/2 & 0 \\ \frac{6\cdot\varepsilon-7}{\varepsilon} & 3 & 0 \\ \frac{-(6\cdot\varepsilon^2-42\cdot\varepsilon+49)}{2\cdot(6\cdot\varepsilon-7)} & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} \varepsilon & 7/2 & 0 \\ \frac{6\cdot\varepsilon-7}{\varepsilon} & 3 & 0 \\ \frac{-(6\cdot\varepsilon^2-42\cdot\varepsilon+49)}{2\cdot(6\cdot\varepsilon-7)} & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

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(Due to screen size limit of TI, the output is cut short. We have to scroll down to get the rest of the matrix. The second snapshot is the result of doing so.)
Letting epsilon approach zero from the, say, positive side, we'll get the following sign column

[+ + + - + +]'

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[2] Paragraph excerpt from Nise, *Control Systems Engineering*.

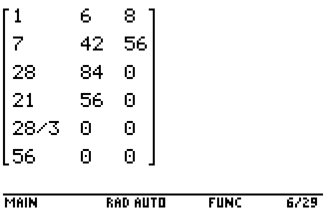
whence two poles in the RHP.

Example. All-0-row

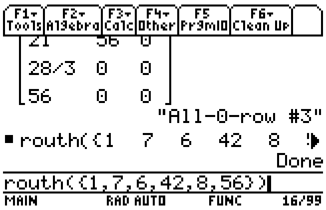
For a system with the following characteristic equation,

s^5+7s^4+6s^3+42s^2+8s+56

The program gives,



I'm using Bestview[3] to devote the whole screen to the Routh table and give you a better view of the matrix. If you actually run the program you'd get,



Notice the quoted line right below the matrix; it says "All-0-row #3" which means row number 3 (row #3) has been an all zero row. What we have in its place now is the derivative of the row immediately above it which is the row containing 7 42 56 as its elements. If we formed an auxiliary polynomial using these elements as coefficients and applied this rule that this polynomial starts with the power of s in the label column[4] and continues by skipping every other power of s, we would be left with

P(s) = 7s^4+42s^2+56

dP(s)/ds = 28s^s+84s^2+0

which latter polynomial's coefficients you see in the row #3 instead of zeros. The remainder of the table is formed in a straightforward manner based on the basic Routh method.

"Let us look further into the case that yields an entire row of zeros. An entire row of zeros will appear in the Routh table when a purely even[5] or purely odd polynomial is a factor of the original polynomial. Even polynomials only have roots that are symmetrical about the origin. This symmetry can occur under several conditions of root position: (1) the roots are symmetrical and real (2) the roots are symmetrical and imaginary (3) the roots are symmetrical and quadrantal." Thus the row of zeros tells us of the existence of an even polynomial whose roots are symmetric about the origin. Some of these roots could be on the jw axis. On the other hand, since jw roots are symmetric about the origin, if we do not have a row of zeros, we cannot have jw roots.

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[3] BestView is developed by Samuel Stearley.

[4] Label column is not shown in the matrix derived by this program; it contains powers of s starting from the highest and ending in zero. Highest power in case of the polynomial in this example is 5, hence the row behind it would have as its label column the power 4. Anyways, note that I'm just explaining the derivation process for those interested in it. You don't have to go through this process. It's been taken care of in the program.

[5] An even polynomial has only even powers of its variable.

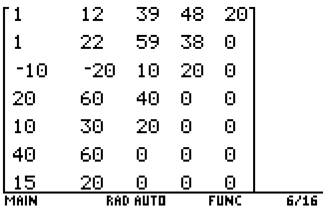
Another characteristic of a Routh table with a row of zeros is that "the row previous to the row of zeros contains the even polynomial that is a factor of the original polynomial. Finally everything from the row containing the even polynomial down to the end of the Routh table is a test of only the even polynomial."

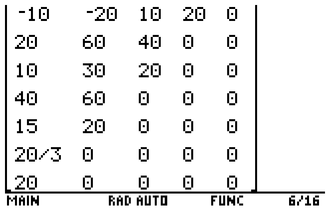
For the characteristic polynomial,

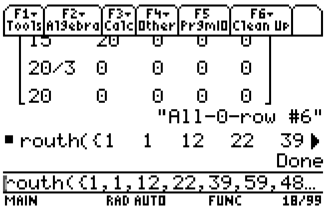
s^8+s^7+12s^6+22s^5+39s^4+59s^3+48s^2+38s+20

tell how many poles are in the RHP, LHP and on the jw axis.

Call the program Routh({1,1,12,22,39,59,48,38,20})







Since the quotation says "All-0-row #6", the row with the elements 40 60 0 0 0 has been recasted by the method just described and the row just above it (10 30 20 0 0) contains the coefficients to an even polynomial. That row is row number 5, the highest power present in our characteristic polynomial is 8 which starts the first row (row #1) so the row #5 starts with s^4. everything from this row down to the end of the table is a test of the following polynomial,

10s^4+30s^2+20

"No sign changes exist from the s^4 row down to the s^0 row. Thus the even polynomial does not have RHP poles. Since there are no RHP poles, no LHP poles are present because of the requirement for symmetry. Hence the even polynomial must have all four of its poles on the jw axis. jw axis poles should have unit multiplicity for stability. For this case, the existence of multiple jw axis poles *would* lead to a perfect fourth order square polynomial. Since s^4+3s^2+20 is not a perfect square, the four jw axis poles are distinct. For other cases the even polynomial must be checked for multiple jw axis poles. The remaining roots of the total polynomial are evaluated from the 1^{st} (s^8) row down to the s^4 row. We notice two sign changes. Thus the rest of the polynomial must have two RHP roots."[6]

	Even (4^{th} order)	Rest (8-4)=4^{th} order	Total (8^{th} order)
RHP	0	2	0+2=2
LHP	0	4-2 =2	0+2=2
jw	(4-0*2)=4	0	4+0=4

[6] ibid.

Acknowledgments

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Rusty Wagner	for VTI.
Kevin Kofler	for AutoAlphaOff, and AutoCloseBracket.
Samuel Stearley	for Ez Greek, BestView, Complete and **copyto_h**. I've used this last one in my program. Refer to his **copyto_h.txt** for more info.

Any suggestions or comments are highly welcome. Feel free to write to OmidLink@hotmail.com.